647510 **BSCmaMN353A-6C** **Seat No :________**

B.Sc. Semester - 6 (NEP-2020) Examination
March/April-2026
MATH: Applied Mathematics-3 (Graph Theory)
(For students having Major subject as Mathematics) (Minor-6)

Time: 2:00 Hours **Marks: 50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Answer any TWO: (10)

(1) Define the degree of a vertex. Prove that the sum of the degrees of all vertices in a graph is equal to twice the number of edges

(2) Explain the union and intersection operations on two graphs G_1 & G_2.

(3) Prove that a graph is disconnected if and only if its vertex set Vcan be partitioned into two non-empty subsets V_1 & V_2 such that no edge exists between a vertex of V_1 and a vertex of V_2.

Que.2 Answer any TWO: (10)

(1) Define: Tree
Prove that in a tree graph T with n vertices is having exactly $n-1$ edges.

(2) Define a binary tree.
Prove that the number of internal vertices in a binary tree is one less than the number of pendant vertices.

(3) Define: (1) Vertex Connectivity (2) Edge Connectivity
Prove that the vertex connectivity of any graph G can never exceed the edge connectivity of it.

Que.3 Answer any TWO: (10)

(1) Define: Planar Graph.
Prove that Kuratowski's second graph $K_{3,3}$ is not planar.

(2) State the Euler's formula for a connected planar graph.
In a simple connected planar graph G, with f regions, n vertices and e edges, prove that (i) $e \geq \frac{3}{2}f$ (ii) $e \leq 3n-6$.

(3) Define: Decyclization.
By taking suitable graph, discuss decyclization of it using Boolean Algebra.

Que.4 Answer any TWO: (10)

(1) Define: Incidence matrix.
Prove that the rank of the incidence matrix $A(G)$ of a connected graph G with n vertices is $n-1$.

(2) Define: (i) Adjacency Matrix (ii) Path Matrix
List atleast five properties of any one of these matrices.

(3) Prove that the set of circuit vectors corresponding to the set of fundamentals circuits with respect to any spanning tree forms a basis for the circuit subspace W_Γ.

Que.5 Answer any TWO: (10)

(1) Prove that a subgraph g of a connected graph G is contained in some spanning tree of G if and only if g contains no circuit.

(2) Prove that any circuit has an even number of edges in common with any cut-set in any connected graph G.

(3) Define: Dominating Set.
By taking suitable example , explain the procedure of finding all minimal dominating sets in a given graph G.

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